Dynamic Stochastic General Equilibrium Framework for Long-term Transportation Policy Analysis

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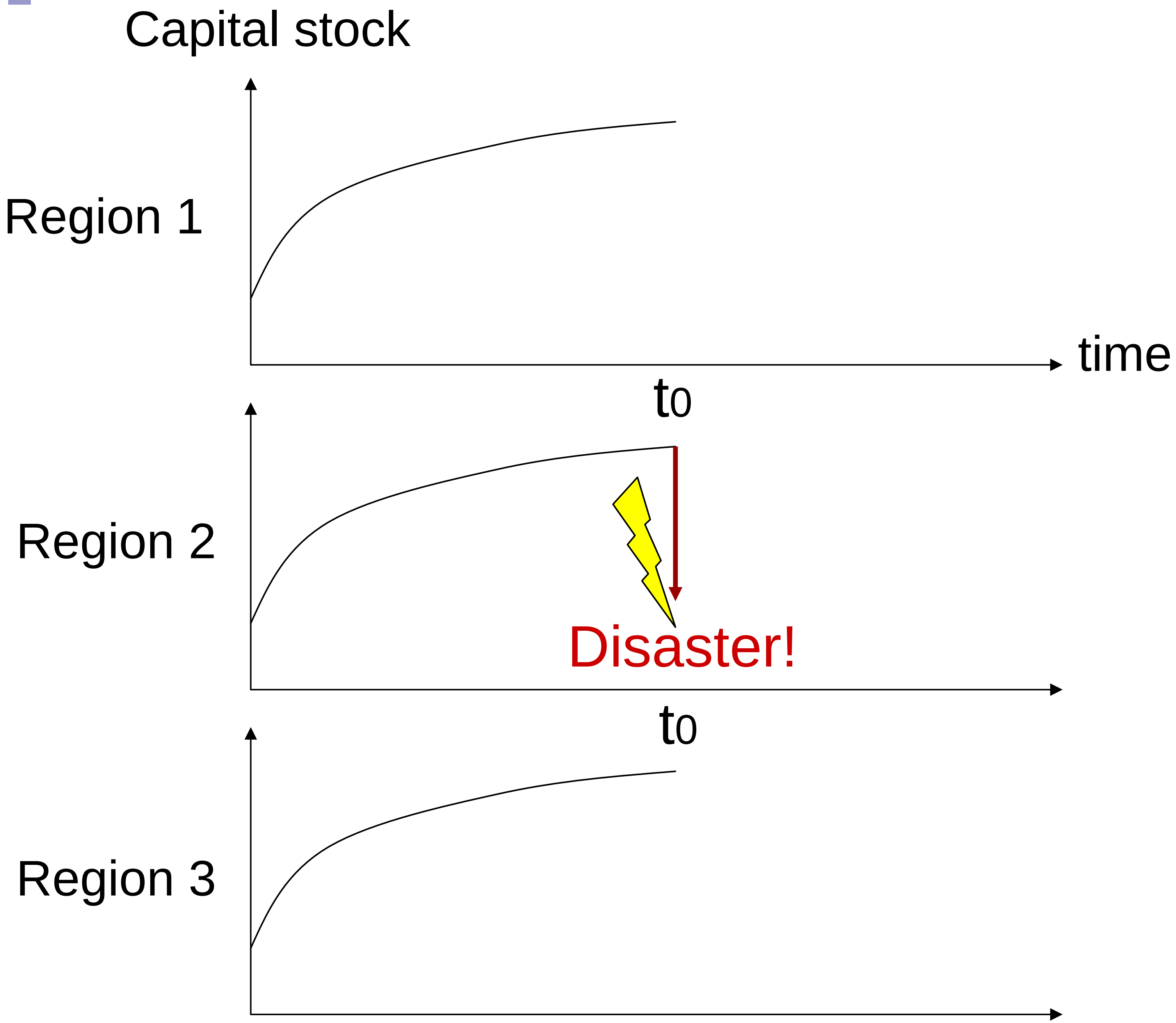
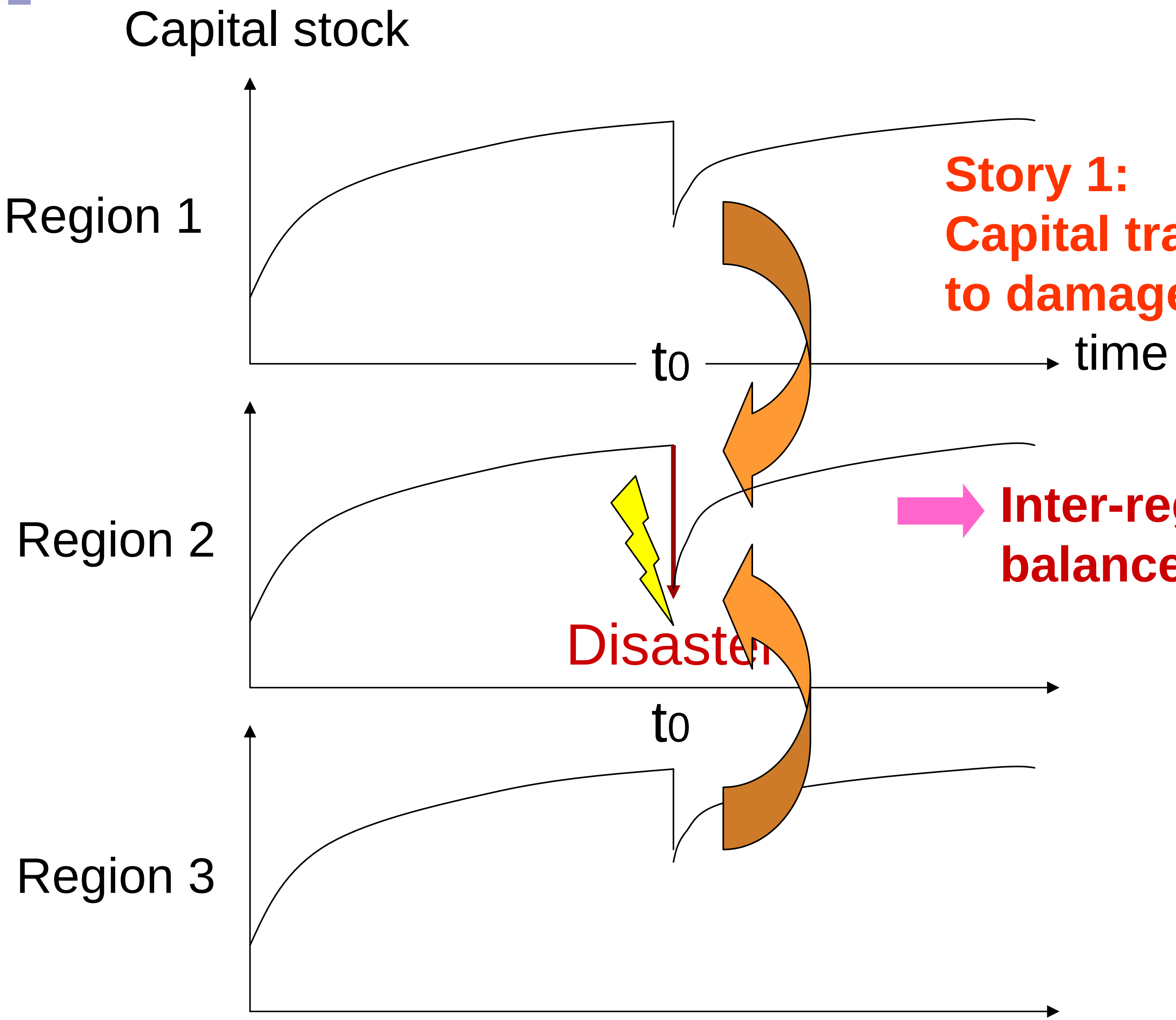
IDRiM2009, Kyoto, Japan, Oct. 12-16, 2009

Focus of the study with Stochastic Dynamic Spatial CGE Approach

Concern

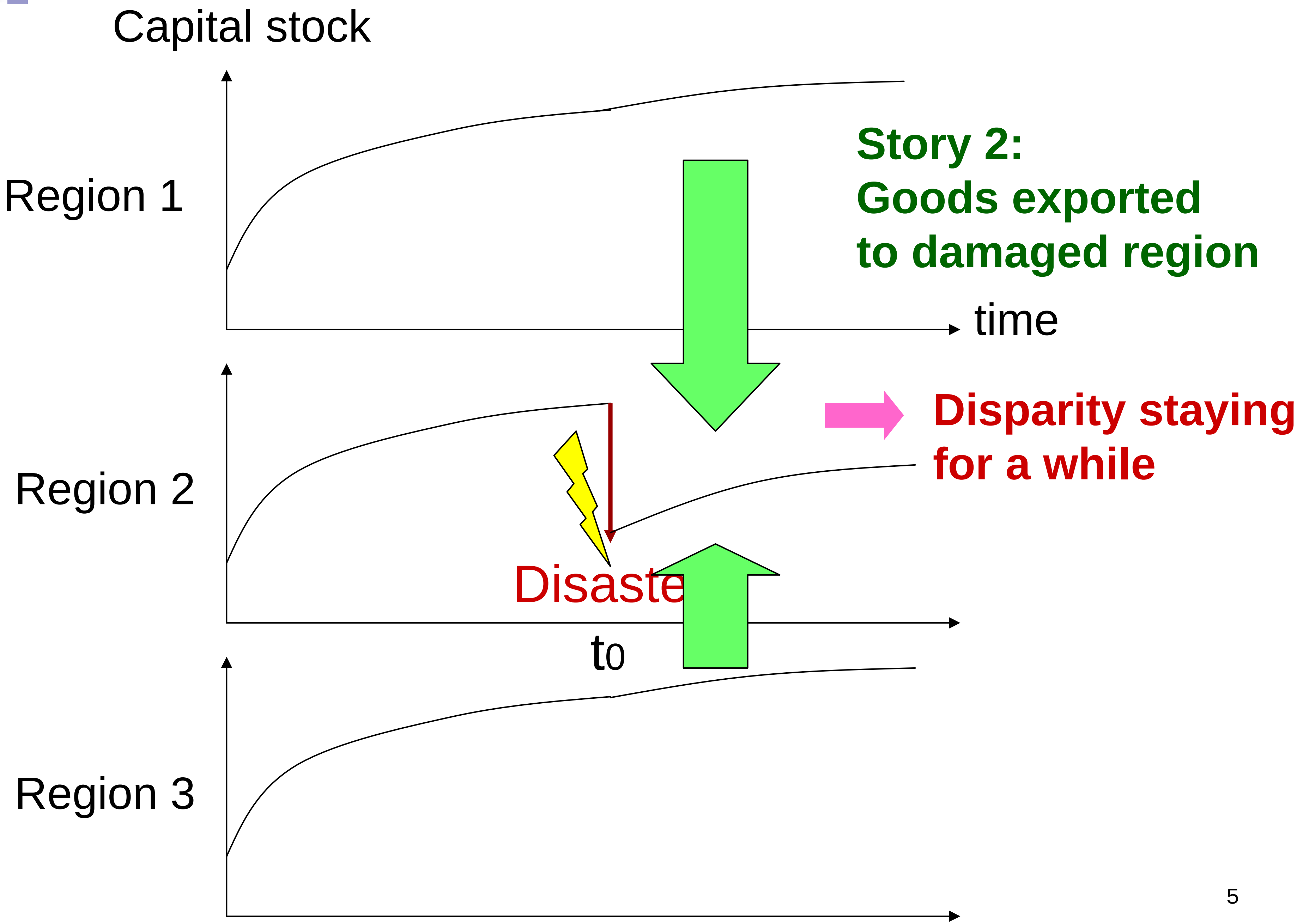
- Disaster as spatial risk that randomly hits region and destroys stocks of local capital and infrastructure
- Roles of transportation sectors and infrastructure in recovery process after local disaster

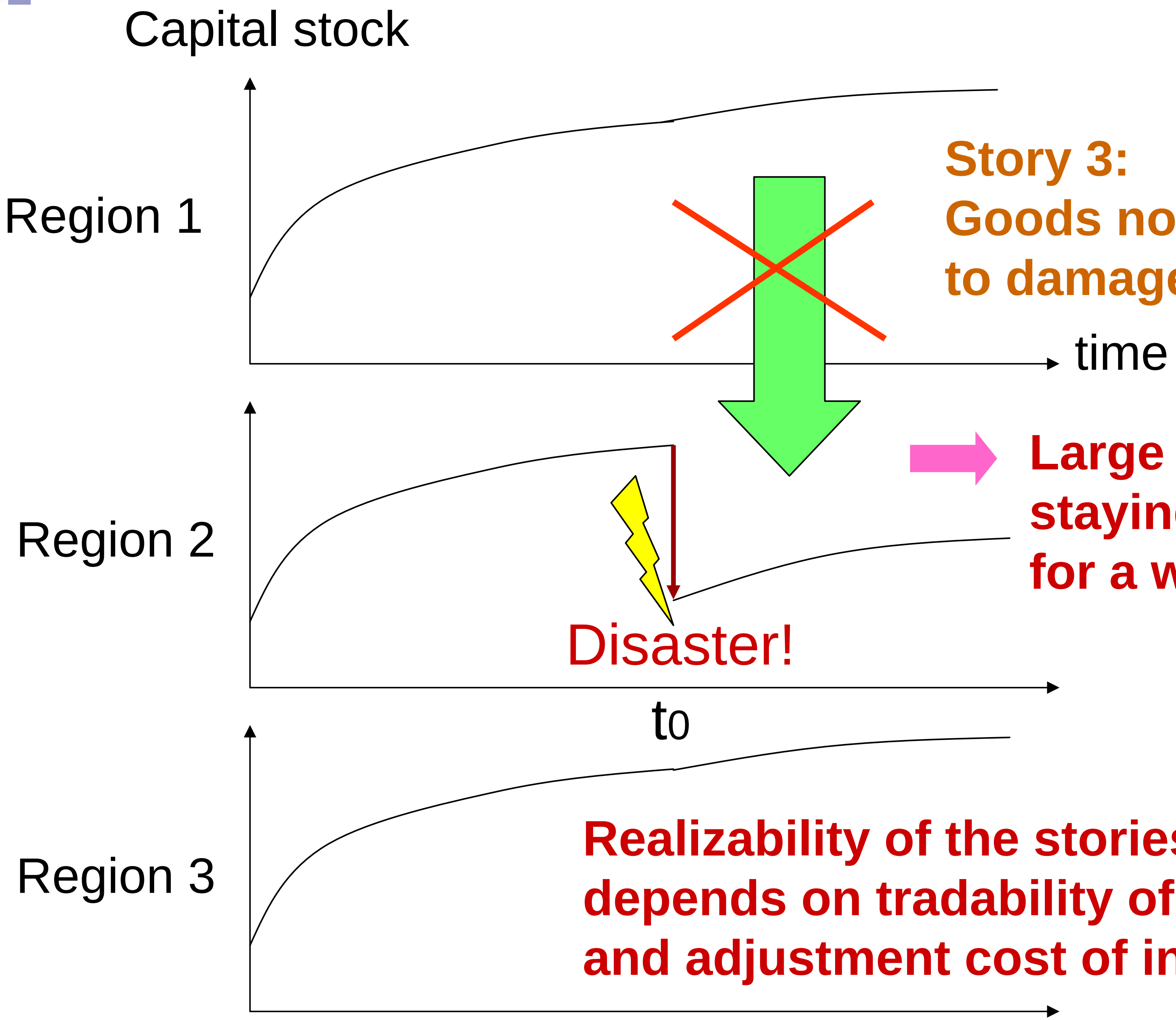

Capital stock
Region 1
time
t0
Region 2
Disaster!
t0
Region 3



Story 1:
Capital transfer
to damaged region

Inter-regionally
balanced recovery


Capital stock
Region 1
Region 2
Region 3
time
Story 2:
Goods exported
to damaged region
Disparity staying
for a while
Disaste
t0

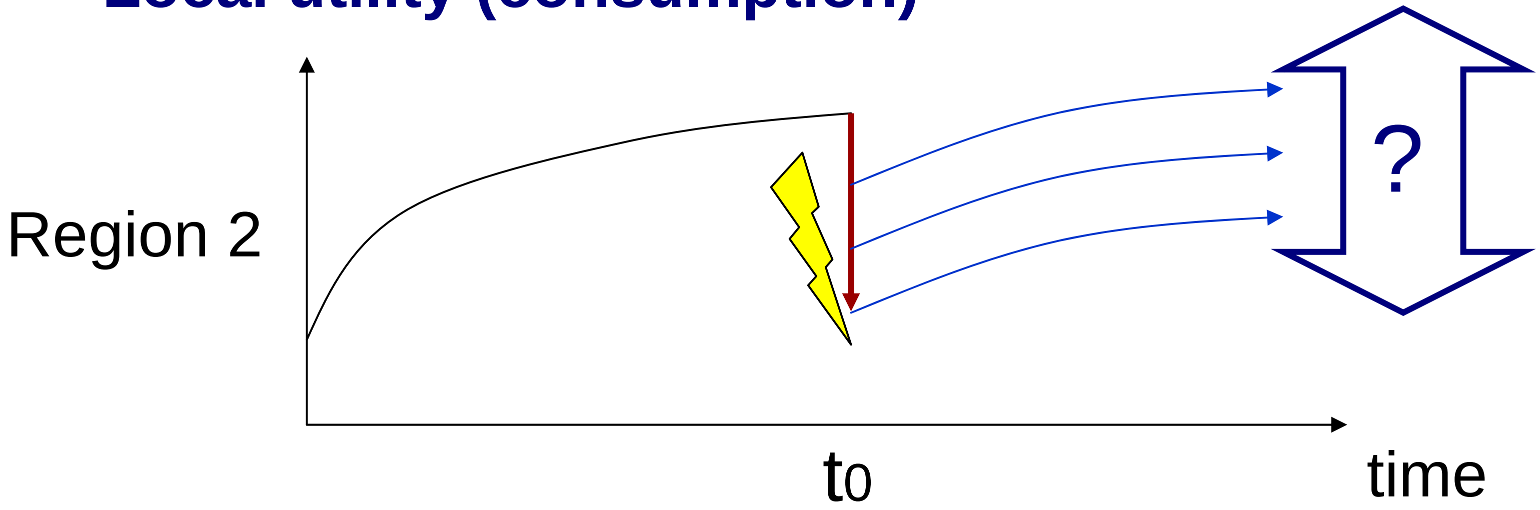


Story 3:
Goods not exported to damaged region

⇒ **Large disparity staying for a while**

Realizability of the stories depends on tradability of goods, and adjustment cost of investment.

Local utility (consumption)



Consumption path, i.e. availability of goods, depends on transportation environment.

Focus of the study with Stochastic Dynamic Spatial CGE Approach (cont'd)

Why "Dynamic"?

- In recovery process after destruction of local "stocks", available goods in a damaged region should be rationally allocated between consumption and reconstruction.

Why "Spatial"?

- In recovery process, goods and service are imported into the damaged region.
- Policy of recovering inter-regional transportation infrastructure is aimed at costless inflow of products of other regions.

Focus of the study with
Stochastic Dynamic Spatial CGE Approach (cont'd)

Why "Stochastic"?

- Local capital is priced under spatial risks, and transportation infrastructure (T-infra.) helps inter-regional risk sharing.
- Do households invest in their region or other regions?

Model
-Environment of the economy

- Dynasty model with continuous time framework
- Real economy with four sectors on agriculture, manufacture, service and transportation
- Small country in the world and M regions in the country
- Constant-returns-to-scale technologies and perfect competition in each private sector
- Armington structure for compounding same goods of different growing districts

Environment of the economy (cont'd.)

- No population (labor) growth
- No technological progress
- Capital market open in the country and closed to foreign country
- Labor market open among sectors in each region while inter-regionally closed
- T-infra. as input of transportation service
- Disaster destroying capital and T-infra. in a region

Four sectors

	1. Agriculture	2.Manufacture	3.Service	4.Transportation
Inputs	Labor (L), Capital (K), Land(B): fixed Armington Intermediate goods (AIG)	L, K, AIG	L, K, AIG	L, K, AIG Trans. Infra. (G)
Outputs	Demanded by Arm-firms → Consumption, IG	Demanded by Arm-firms → Consumption IG, Capital	Demanded by Arm-firms → Consumption IG	Demanded by Arm-firms
Inter-national trade	Tradable	Tradable	Non-tradable, endogenous domestic price	

Production Technologies

- Agricultural sector (j=1) in region m (=1,･･･,M)

$$Y_1^m = \min\left[F_1^m(L_1^m, K_1^m, B^m), \frac{X_{11}^m}{a_{11}^m}, \frac{X_{21}^m}{a_{21}^m}, \frac{X_{31}^m}{a_{31}^m}\right]$$

Labor Capital Land

Y_j^m : Production of sector j in region m

X_{ij}^m : Demand for Armington intermediate “i” goods for production of sector j in region m

$$F_1^m(L_1^m, K_1^m, B^m) = a_1^m (L_1^m)^{\alpha_{11}^m} (K_1^m)^{\alpha_{12}^m} B^{\alpha_{13}^m}$$

$$(\alpha_{11}^m + \alpha_{12}^m + \alpha_{13}^m = 1)$$

Production Technologies (cont'd.)

- Manufacturing (j=2) and service (j=3) sector in region m (=1,・・・,M)

$$Y_j^m = \min\left[F_j^m(L_j^m, K_j^m), \frac{X_{1j}^m}{a_{1j}^m}, \frac{X_{2j}^m}{a_{2j}^m}, \frac{X_{3j}^m}{a_{3j}^m}\right]$$

$$F_j^m(L_j^m, K_j^m) = a_j^m (L_j^m)^{\alpha_{j1}^m} (K_j^m)^{\alpha_{j2}^m}$$

$$(\alpha_{j1}^m + \alpha_{j2}^m = 1)$$

Production Technologies (cont'd.)

- Transportation sector (j=4) in region m (=1,・・・,M)

Inter-regional transportation from "m" to "n (=1,・・・,M)" is supplied by firms in "m" (origin).

$$Y_4^m = \min\left[F_4^m(L_4^m, K_4^m, G), \frac{X_{14}^m}{a_{14}^m}, \frac{X_{24}^m}{a_{24}^m}, \frac{X_{34}^m}{a_{34}^m}\right]$$

G : Inter-regional transportation infrastructure

Production Technologies (cont'd.)

$$F_4^m(L_4^m, K_4^m, G) = a_4^m (L_4^m)^{\alpha_{41}^m} (K_4^m)^{\alpha_{42}^m} G^{\alpha_{43}^m}$$

Cobb-Douglas

$$(\alpha_{41}^m + \alpha_{42}^m = 1)$$

Hicks neutral

$$(\alpha_{42}^m + \alpha_{43}^m < 1)$$

Transportation service for imported goods are provided by foreign firms with a constant fee.

Production Technologies (cont'd.)

- Armington firms for j-goods (j=1,2,3)
  in region m (=1,･･･,M)

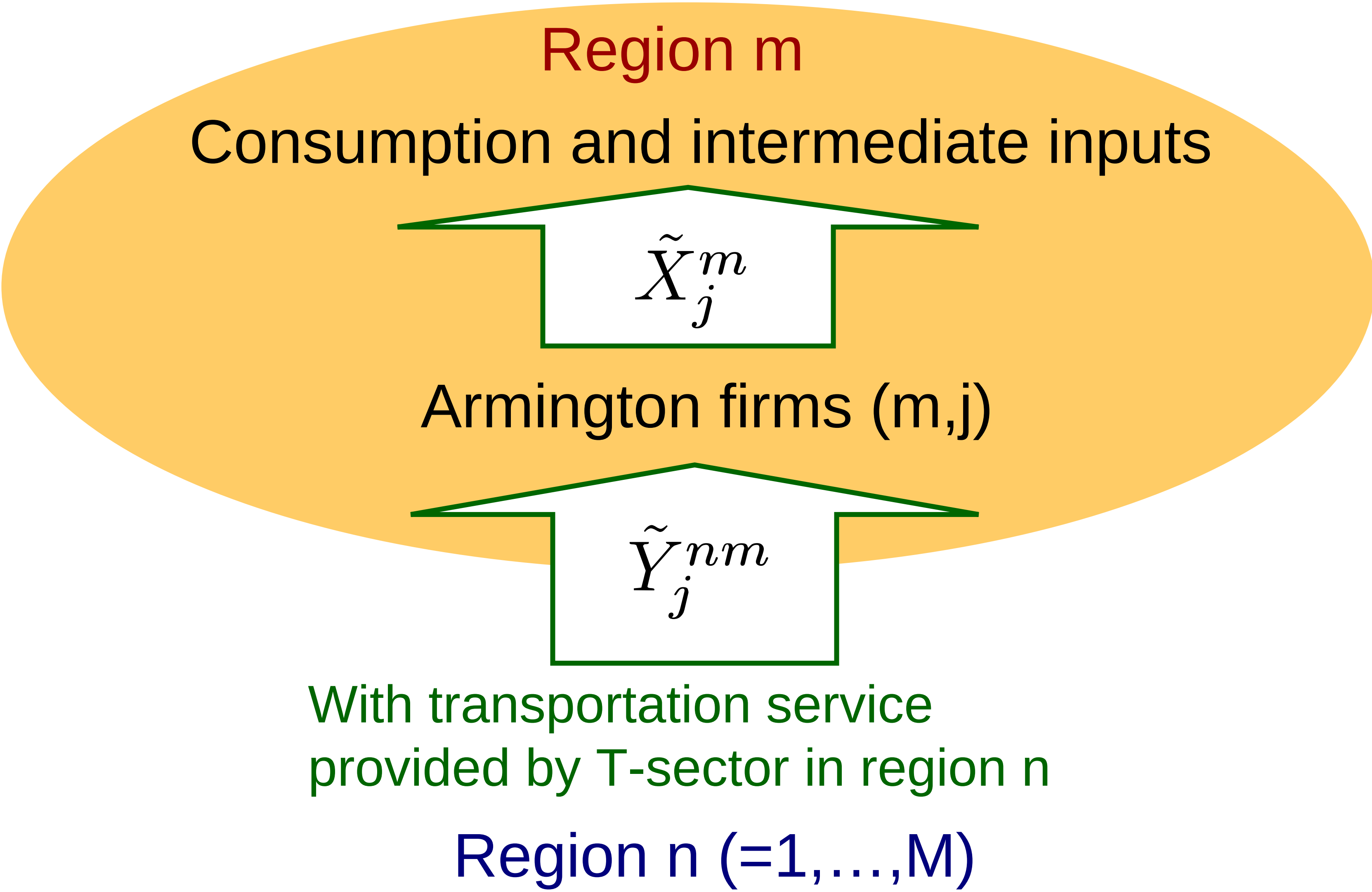
  Armington firms (m, j) demand transportation service when they import original goods produced in other regions.

$$\tilde{X}_j^m = \left[\sum_n \gamma_j^m (\tilde{Y}_j^{nm})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

$\tilde{X}_j^m$: Production of Arm-goods in (m, j)

$\tilde{Y}_j^{nm}$: Arm-firms' demand for original j goods produced in region n

Transportation and Armington firms



Armington firms' problem

Arm-firm (m,j)'s cost minimization

$$\min \sum_n p_j^{nm} \tilde{Y}_j^{nm}$$

Subject to

$$H^m(\tilde{Y}_j^{1m}, \tilde{Y}_j^{2m}, \cdots) = \left[\sum_n \gamma_j^m (\tilde{Y}_j^{nm})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} = \tilde{X}_j^m$$

$$p_j^{nm} = p_j^n + \underline{\hat{p}_j^{nm}}$$

Price for transporting one unit of original goods j from region n

Factor Markets

Labor (L)

- Closed in each region (no migration)
- Inter-sectorally mobile, every period
- Inelastic total supply

Capital (K)

- Closed in the country
- Inter-regionally and inter-sectorally mobile, every period
- Mnf-goods used for investment

  Possible to import Mnf-goods and invest in capital.

Factor Markets (cont'd.)

Land (H)

- Fixed and constant throughout.
- Used only by Ag-sector.
- Owned by local households.

Development of infrastructure

- Jump process associated with disaster attack

$$dG = (I - \delta G)dt - G(\frac{\psi^1}{2}dq^1 + \frac{\psi^2}{2}dq^2)$$

G : Stock of Inter-regional T-infra.

I : Investment

δ : Depreciation rate

$0 < \psi^m < 1$: Damage ratio in region m (m=1,2)

q^m : Poisson jump process in region m (m=1,2)

Development of infrastructure (cont'd)

- Poisson jump process in region m

$$dq^m = 0 \quad \text{with prob. } 1 - \lambda^m dt$$
$$= 1 \quad \text{with prob. } \lambda^m dt$$

- Cost for investment with adjustment cost

$$I\left\{1 + \Psi\left(\frac{I}{G}\right)\right\}$$

$$\Psi(0) = 0,\ \ \Psi'(\cdot) > 0,\ \ 2\Psi' + \frac{I}{G}\Psi'' > 0$$

Households in each region

- Homogeneous
- Local population normalized to be one
- Perfect foresighted on infinite time horizon

Instantaneous utility

$$u(\nu_1^m(t), \nu_2^m(t), \nu_3^m(t)) = (\nu_1^m(t))^{b_1}(\nu_2^m(t))^{b_2}(\nu_3^m(t))^{b_3}$$

Ag-goods **Mnf-goods** **Service**

$$(b_1 + b_2 + b_3 = 1)$$

$\nu^m(t)$:Level of the indirect utility

Households in each region (cont'd)

Present value of the expected utility

$$
\begin{aligned}
EU^m &= E\int_0^\infty U(\nu^m(t))e^{-\rho t}dt \\
&= E\int_0^\infty \frac{\nu^m(t)^{1-\theta}-1}{1-\theta}e^{-\rho t}dt
\end{aligned}
$$

Social welfare function

$$W = EU^1 + EU^2$$

Event sequence (market equilibrium) : 7 steps in one cycle.

1) A set of state variables, $(a(t), \underline{G(t)})$, is valuated.

Vector of T-infra.

2) A set of taxation, $\tau(t)$, is given.

3) Household determines demands ν_j^m, and supplies labor and capital that firms demand. Production is implemented, and products are transacted and consumed.

→ Market equilibrium

(Every price is determined.)

Event sequence (cont'd.)

4) Investment in T-infra. is carried out, while investment goods will be embodied into the stock after step 6).

5) Factor payment, $(w_j^m l_j^m, r_j^m k_j^m, \pi_2^m h^m)$, is done.

(Next instant of time, t+dt)

6) Disaster arrives and destroys capital and T-infra.

7) Capital, k_j^m or $(1-\psi^m)k_j^m$, is returned to owners.

Firms and households' intra-temporal behavior (market equilibrium)

- Rent maximization by Ag-sector
- Cost minimization by Mnf-sector, Serv-sector, Trans-sector and Arm-firms
- Expenditure minimization by households
- Market clears every time, and firms do not reserve any profit and debt. Hence inter-temporal problems reduce only to a problem of households in each region.

First-best problem

$$\begin{aligned}\max\ W &= EU^1 + EU^2 \\ &= E\int_0^{\infty}\{U^1(\nu^1(t)) + U^2(\nu^2(t))\}e^{-\rho t}dt\end{aligned}$$

Transition of state variables

(Arm.) Mnf-goods clearing

$$dA^m = \{\tilde{X}_2^m - \nu_2^m - T^m - \sum_j a_{2j}^m Y_j^m\}dt$$

$$- \sum_n \psi^n (\sum_j \underline{K_j^{mn}}) dq^n \qquad \text{for m=1,2}$$

Capital invested from m to n

$$dG = (I - \delta G)dt - G(\frac{\psi^1}{2}dq^1 + \frac{\psi^2}{2}dq^2)$$

First-best problem (cont'd)

Current value function

$$V(A^1(t), A^2(t), G(t))$$

Stochastic Hamilton-Jacobi-Bellman equation

$$\rho V \quad = \quad U^1 + U^2 + \{(\mathrm{b} + \underbrace{\sum_n \lambda^n \mathrm{l}^n}_{\text{Expected losses}})^T \nabla\} V + \underbrace{\frac{1}{2} \sum_n \lambda^n \{\mathrm{l}^{nT} \nabla\}^2 V}_{\text{Related with risk aversion}}$$

where

$$\nabla := \left(\frac{\partial}{\partial A_1}, \frac{\partial}{\partial A_2}, \frac{\partial}{\partial G}\right)$$

$$\mathbf{b} = \begin{pmatrix} \tilde{X}_2^1 - \nu_2^1 - T^1 - \sum_j a_{2j}^1 Y_j^1 \\ \tilde{X}_2^2 - \nu_2^2 - T^2 - \sum_j a_{2j}^2 Y_j^2 \\ I - \delta G \end{pmatrix} \qquad \mathbf{l}^n = \begin{pmatrix} -\psi^n \sum_j K_j^{1n} \\ -\psi^n \sum_j K_j^{2n} \\ -\frac{\psi^n G}{2} \end{pmatrix}$$

Other main constraints

(Arm-) Ag-goods and service clearing

$$\tilde{X}_j^m = \underset{\uparrow}{\nu_j^m} + \underbrace{\sum_i a_{ji}^m Y_i^m}_{} \qquad \text{(for j=1,3, m=1,2)}$$

consumption **intermediate input**

Original goods clearing

$$Y_j^m = \sum_n \tilde{Y}_j^{mn} + EX_j^m \qquad \text{(for j=1,2,3, m=1,2)}$$

demand by Arm-firms in n **Export for j=1,2**

Transportation service clearing

$$Y_4^m = \sum_j \beta_j \tilde{Y}_j^{mn} + \sum_j \beta_j^f EX_j^m \quad \text{(for m=1,2, m}\neq\text{n)}$$

for export>0

Other main constraints (cont'd)

Investment in T-infra

$$T^1 + T^2 = I\left\{1 + \Psi\left(\frac{I}{G}\right)\right\}$$

Labor and capital supply

$$\sum_j L_j^m = 1 \qquad \sum_n \sum_j K_j^{mn} = A^m$$ (for m=1,2)

Capital demand

$$K_j^n = \sum_m K_j^{mn}$$ (for n=1,2, j=1,…,4)

First-order optimal conditions

Labor pricing conditions

$$\zeta^m_{Y^j} \frac{\partial F^m_j}{\partial L^m_j} = \mu^m_L$$

Wage: Inter-sectorally identical in m

(for j=1,…,4)

Value added price of org. goods

* Notation

$$V_1 := \frac{\partial V}{\partial A^1}, \quad V_2 := \frac{\partial V}{\partial A^2}, \quad V_3 := \frac{\partial V}{\partial G}$$

$$V_{11} := \frac{\partial^2 V}{\partial (A^1)^2}, \quad V_{12} := \frac{\partial^2 V}{\partial A^1 \partial A^2}, \cdots$$

First-order optimal conditions (cont'd)

Asset pricing conditions in region 1

Not appear in market equilibrium

$$\mu_A^1 = \zeta_{Y\,j}^1 \frac{\partial F_j^1}{\partial K_j^1} - \underbrace{V_1 \lambda^1 \psi^1}_{\text{Expected loss}}$$

$$\underbrace{+\lambda^1(\psi^1)^2 \left[(\sum_j K_j^{11}) V_{11} + \boxed{(\sum_j K_j^{21}) V_{12}} + \frac{G}{2} V_{13} \right]}_{}$$

Risk premium: relative to the sum of capital

(for j=1,…,4 in region 1)

$$= \zeta_{Y\,j}^2 \frac{\partial F_j^2}{\partial K_j^2} - V_1 \lambda^2 \psi^2$$

$$+\lambda^2(\psi^2)^2 \left[(\sum_j K_j^{12}) V_{11} + (\sum_j K_j^{22}) V_{12} + \frac{G}{2} V_{13} \right]$$

(for j=1,…,4 in region 2)

Effective return on capital:
Inter-regionally and inter-sectorally identical

First-order optimal conditions (cont'd)

Inter-regional import of Org. goods: optimal behavior of Arm-firms (n,j)

$$\zeta^{n}_{X\,j}\frac{\partial H^{n}_{j}}{\partial \tilde{Y}^{m\,n}_{j}} = \eta^{m}_{Y\,j} + \eta^{m}_{Y\,4}\beta_{j}$$

Payment for transportation service

Marginal prod. of Arm-goods

Price of Org-goods j in region m

Price of Arm-goods j in region n

Note that Arm-firms do not use labor and capital for production.

First-order optimal conditions (cont'd)

Shadow prices and market prices

$$V_1 = \zeta_{X2}^1, V_2 = \zeta_{X2}^2$$

Shadow price of asset is equalized to market price of Arm-Mnf-goods.

Equalization of shadow prices of regional assets

Optimal taxation condition implies

$$V_1 = V_2 \ (:= V_A)$$

hence

$$\zeta_{X2}^1 = \zeta_{X2}^2.$$

Too strong?

First-order optimal conditions (cont'd)

Optimal investment on T-Infra.

$$\iota := \frac{I}{G}$$ **Investment rate**

$$1 + \Psi(\iota) + \Psi'(\iota) \cdot \iota = \frac{V_G}{V_A} := T_q$$ **Tobin's q**

$$T_q \leq 1 \quad \Rightarrow \quad \iota = 0$$

$$T_q > 1 \quad \Rightarrow \quad \iota > 0$$

Reconstruction decision after disaster is given by the ratio of V_G to V_A (i.e. market price of Mnf-goods). Drastic increase of Mnf-goods price caused by shortage of the production may decrease Tobin's q.

What is investigated?

- We formulated a framework, from which dynamic strategy of representative household in each region is introduced.

- With Monte Carlo simulation, we can describe disparity (or balance) of GRP growth among regions after a local hit of disaster.

What is investigated? (cont'd)

- We can analyze inter-regional risk sharing where T-infra. works partially for diversification. Since households are immobile, households in some region take more risk than ones in other regions.

- We are concerned with game theoretic equilibrium of jurisdictional T-infra. provision. Role of the central government by means of subsidization and provision of national T-infra. is also an object of research interest.

Thank you very much.

Sorry for rushing presentation.